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RECONSTRUCTING THE EFFECTIVE COEFFICIENT OF THERMAL CONDUCTIVITY OF ASBESTOS–TEXTOLITE FROM THE SOLUTION OF THE INVERSE PROBLEM

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The article examines the practical application of the algorithm for solving inverse problems in processing experimental data.

The intense development of the theory and the increasing range of application of the methods of solving inverse problems of heat exchange [1] led to their widespread use in thermophysical research [2-5]. Such an approach in the investigation of the thermophysical characteristics of high-temperature composite materials under nonsteady conditions solves the problem of modeling the structure of the material and the nature of how internal processes proceed [6], and moreover, it makes it possible to determine these characteristics for mathematical models in which their application is assumed.

Sometimes the problem of determining the effective values of thermophysical characteristics may be examined; the use of these characteristics makes it possible to generalize in fairly simple form the results of experimental investigations. Furthermore, such characteristics may be used for calculating temperature fields of coatings in the range of change of external conditions that is of interest to the researcher.

The principal object of the present work consists in investigating the possibility of the practical application of the methods of inverse problems for determining the thermophysical characteristics of composite materials under nonsteady conditions.

We analyze the errors connected with thermocouple temperature measurements in high-temperature decomposing material, and the accuracy of the obtained results is evaluated. For processing the experimental data we used the algorithm for solving inverse coefficient problems of heat conduction explained in [2].

We analyzed a model of an unbounded flat plate in which at four points thermocouple measurements were carried out.

The temperature measurements at the outer points of the examined region were used as thermal boundary conditions. The input data for solving the inverse problem were the temperature measurements at the inner points of the region.

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The inverse problem was formulated in the following manner. We had to determine the functions $\lambda(T)$ and $T(x, \tau)$ from the conditions

$$c(T)\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right),\quad 0<x<b,\ 0<\tau\leqslant\tau_m, \tag{1}$$

$$T(x,\ 0)=T_0,\quad 0\leqslant x\leqslant b, \tag{2}$$

$$T(x_i,\ \tau)=f_i(\tau),\quad i=\overline{1,\ 4},\quad x_1=0<x_2<x_3<x_4=b, \tag{3}$$

where $c(T)$, $f_i(\tau)$, $i = \overline{1,\ 4}$ are known functions.

As objective function we examined the rms discrepancy

$$I=\sum_{i=2}^{3}\int_0^{\tau_m}[T(x_i,\ \tau,\ \lambda(T))-f_i(\tau)]^2\,d\tau, \tag{4}$$

where $T_i(x_i,\ \tau,\ \lambda(T))$, $f_i(\tau)$ are the temperatures at points x_i at the instant τ, calculated by using (1)-(3) and measured, respectively. On account of the homogeneity of (1) the region of determining the function $\lambda(T)$ was fixed in the form of the interval $D = [T_{\min},\ T_{\max}]$.

The sought function $\lambda(T)$ was represented by using cubic B-slines on the grid

$$\omega=\{T_k=T_{\min}+k\Delta T,\quad k=-2,\ -1,\ \ldots,\ m+3;$$
$$\Delta T=(T_{\max}-T_{\min})/m\};\quad \lambda(T)=\sum_{k=-1}^{m+1}\lambda_k B_k(T). \tag{5}$$

Here, $B_k(T) = B_0(T - k\Delta T)$ is the cubic B-spline [7].

In accordance with [2], the boundary problem (1)-(3) was presented in the form of the problem of heating an unbounded multilayered plate with the same thermophysical properties of the layers and zero contact resistance between them. In the case under examination the number of layers was equal to three.

With the aid of the notion (5) the formulated inverse problem reduced to seeking the $(m + 1)$-dimensional vector $\bar{\lambda}=\{\lambda_0,\ \lambda_1,\ \ldots,\ \lambda_m\}$ from the condition of minimum of the functional (4) with the constraints (1)-(3). Minimization of the functional was carried out by the method of conjugate gradients [8]. The approximations of the sought parameters were determined by the formulas

$$\lambda_k^{(p+1)}=\lambda_k^{(p)}+\alpha^{(p)}g_k^{(p)},\quad k=\overline{0,\ m},\quad p=0,\ 1,\ 2,\ \ldots, \tag{6}$$
$$g_k^{(p)}=-I_k^{'(p)}+\beta^{(p)}g_k^{(p-1)},$$
$$\beta_0=0,\quad \beta^{(p)}=\sum_{k=0}^{m}(I_k^{'(p)}-I_k^{'(p-1)})\,I_k^{'(p)}/\sum_{k=0}^{m}(I^{'(p-1)})^2.$$

The expression for the components of the vector of the gradient of the objective functional (4) was obtained with the aid of the solution of the problem conjugate to the initial one, and it had the form

$$I_k'=\sum_{i=1}^{3}\int_{x_i}^{x_{i+1}}\int_0^{\tau_m}\psi_i(x,\ \tau)\left[\frac{\partial^2 T_i}{\partial x^2}\,B_k(T)+\left(\frac{\partial T_i}{\partial x}\right)^2\frac{dB_k(T)}{dT}\right]d\tau dx, \tag{7}$$
$$k=\overline{0,\ m}.$$

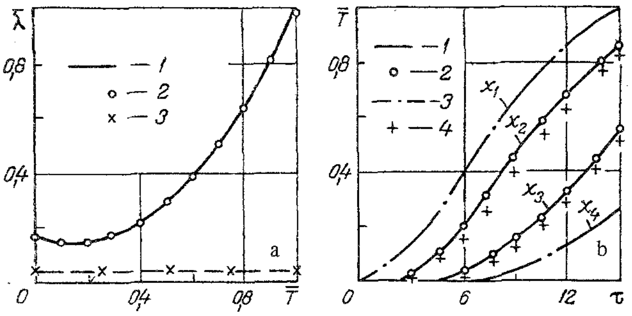

Fig. 1. Solution of the model problem: a) reconstruction of the temperature dependence of the thermal conductivity [1) exact value; 2) reconstructed values; 3) initial approximation]; b) temperature at the points of mounting the thermocouples [1) exact values; 2) reconstructed values; 3) boundary conditions; 4) experimental values]. T_{min} = 323°K, T_{max} = 2159°K, λ_{max} = 1.858 W/(m·°K). τ, sec.

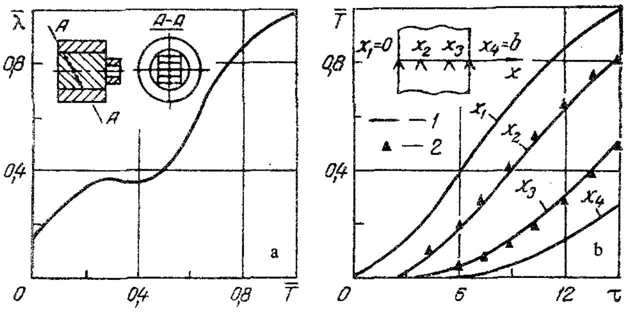

Fig. 2. Processing of the experimental data: a) reconstructed temperature dependence of the thermal conductivity; b) temperature at the points of mounting the thermocouples [1) experimental values; 2) reconstructed values]. T_{min} = 323°K, T_{max} = 2159°K, λ_{max} = 1.033 W/(m·°K).

For the linear evaluation of the depth of descent of α at the p-th iteration we used the solution of the boundary problem for the temperature increment $\vartheta_i(x, \tau)$. In this case in the calculation of α we have from the condition

$$\min_{\alpha} (\overline{\lambda}^{(p)} + \alpha^{(p)} \overline{G}^{(p)}), \text{ where } \overline{G}^{(p)} = \{g_k^{(p)}\}, \quad k = \overline{0, m},$$

$$\alpha^{(p)} = -\frac{\sum\limits_{i=2}^{3} \int\limits_0^{\tau_m} [T_i(x_i, \tau, \overline{\lambda}^{(p)}) - f_i(\tau)]\, \vartheta_i(x, \tau)\, d\tau}{\sum\limits_{i=2}^{3} \int\limits_0^{\tau_m} [\vartheta_i(x, \tau)]^2\, d\tau}. \quad (8)$$

All boundary problems were solved numerically by using the monotonic implicit approximation procedure [9]. The calculations were carried out with the difference grid $n_x \cdot n_\tau$ = 41·51.

The results of non-steady-state temperature measurement were processed in three stages: mathematical modeling for evaluating the certainty of the reconstructed thermal conductivity of asbestos textolite with the thermocouples as arranged in the experiment, processing of the experimental data, and evaluation of the accuracy of the obtained results when there were errors of the measured temperatures.

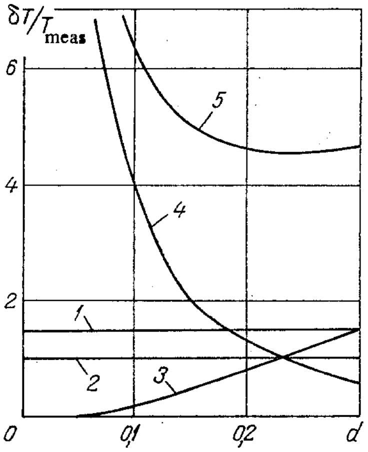

Fig. 3

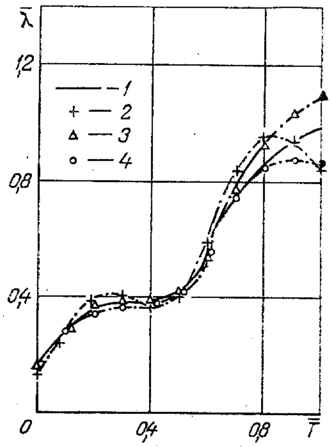

Fig. 4

Fig. 3. Dependence of the error of temperature measurement inside the specimen on the diameter of the thermoelectrode: 1) calibration of the loop and interpretation of the oscillogram; 2) calibration of the thermocouples; 3) distortion of the temperature field by the thermocouple; 4) shunting of the thermocouple by an electrically conducting coke layer; 5) total error. Thermocouple VR5/VR20, l/d = 150, $dT/d\tau$ = 150°K/sec, T_{be} = 1800°K, T_{meas} = 2300°K. $\delta T/T_{meas}$, %; d, mm.

Fig. 4. Evaluation of the reconstructed dependence of the thermal conductivity: 1) nominal value; 2) distortion of the temperature with normal distribution ($\varepsilon_{max} = \pm 0.07$ T); 3) $x_2 = 0.65 \cdot 10^{-3}$ m, $x_3 = 1.45 \cdot 10^{-3}$ m; 4) $x_2 = 0.55 \cdot 10^{-3}$ m; $x_3 = 1.55 \cdot 10^{-3}$ m (λ_{max} = 1.033 W/(m·°K)).

In the mathematical modeling the exact values of the "measured" temperatures $f_i(\tau)$, i = 2, 3 were obtained from the solution of the direct problem (1)-(3) for which we used as boundary conditions the experimentally measured temperatures at the points $x_1 = 0$ and $x_4 = 2 \cdot 10^{-3}$ m. The calculated temperature dependences at the points where the thermocouples were mounted ($x_2 = 6 \cdot 10^{-4}$ m, $x_3 = 1.5 \cdot 10^{-3}$ m) were the input data for solving the inverse problem of determining the "unknown" function $\lambda^*(T)$. As the exact value of the reconstructed thermal conductivity we considered the polynomial

$$\lambda^*(T) = 5.95 \cdot 10^{-7} T^2 - 6.33 \cdot 10^{-4} T + 0.423 \ [\text{W/m} \cdot \text{deg K}]$$

In the calculations we used the temperature dependence of the coefficient of volumetric heat capacity of asbestos textolite presented in [6].

Figure 1 shows the exact dependences and the dependences reconstructed from the solution of the inverse problem in solving the model example: $\lambda(T)$ and $T(x_i, \tau)$, i = 2, 3. The number of sections of the spline-approximation was taken as four. For the sake of comparison, Fig. 1 presents the experimental temperature values at the points where the thermocouples are mounted. The results of processing the experimental data are shown in Fig. 2.

The temperature measurements were carried out in a model of asbestos—textolite; the design of its working part is shown diagrammatically in Fig. 2. High-temperature tungsten—tungsten-rhenium thermocouples VR5/VR20, with diameter $d = 1 \cdot 10^{-4}$ m, butt-welded in an inert atmosphere, were placed in the plane of the oblique section of the working part of the model.

Evaluations of [10] showed that it is preferable to use thermocouples without electrically insulating coating because the use of presently available coatings at temperatures T > 1800°K is inefficient and causes considerable distortion of the temperature field in the specimen.

The ratio of the length of the isothermal part of the thermocouple to its diameter was $l/d = 150$. The elements of the model were glued together with phenol resin, and then the coordinates of embedding the thermocouples were determined from x-ray photographs. To prevent heat removal, the lateral surface of the working part of the model was heat-insulated.

In the analysis of the accuracy of temperature measurements in the experiment, we used the results explained in [10].

Figure 3 shows the magnitudes of the errors of measuring the temperature of 2300°K in high-temperature composite material in dependence on the diameter of the thermoelectrode for the thermocouple VR5/VR20. The same figure also gives the total error of temperature measurement, which for the experiment under consideration may amount to 6-7%.

To evaluate the possible magnitude of the error of the temperature dependence of the effective coefficient of thermal conductivity of Textolite, reconstructed from the solution of the inverse problem, mathematical modeling was carried out. As the nominal thermal conductivity we used the dependence $\lambda(T)$ obtained in processing the experimental results; as input data we used the reconstructed values of the temperatures at the points of mounting the thermocouples. The errors in the experimental functions $f_i(\tau)$, $i = 2, 3$, were simulated by a random number transducer with normal distribution of the probability density of the distortions with an error not exceeding 7% of the actual temperature. The results of the modeling are presented in Fig. 4.

Since in the analysis of the errors of temperature measurements we did not take into account any possible errors in determining the coordinates of the thermocouple mounting, we evaluated the maximum possible deviation of the temperature dependence of the thermal conductivity of asbestos–textolite from the one obtained in processing the experimental results. It was assumed that the errors in determining the position of the thermocouples amount to $\Delta x_i = \pm 5 \cdot 10^{-5}$ m and that they have different signs. As input data we used the experimentally measured temperatures $f_i(\tau)$, $i = 2, 3$. The obtained results are presented in Fig. 4.

Processing of the experimental data and an analysis of the accuracy of the obtained results showed that the approach based on solving inverse coefficient problems of heat conduction may be used successfully in processing the results of real thermophysical experiments.

NOTATION

c, volumetric heat capacity; λ, thermal conductivity; T, temperature; x, space coordinate; τ, time; τ_m, b, right-hand boundary value of time and space intervals, respectively; $f_i(\tau)$, input temperatures; I, functional; θ, temperature increase; ψ, conjugate variable; $\overline{T} = (T - T_0)/T_{max} - T_{min})$, dimensionless temperature, $\overline{\lambda} = \lambda/\lambda_{max}$, dimensionless thermal conductivity; n_x, n_τ, number of nodes in difference approximation with respect to the space and time coordinates, respectively; α, β, parameters of the method of conjugate gradients; p, number of iteration. Subscripts: min, minimal; max, maximal value; 0, initial value; be, beginning of electrical conductance; meas, measured value.

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CONSTRUCTION OF EXPLICIT FUNCTIONS FOR DETERMINING THE COEFFICIENTS OF INTERNAL HEAT AND MASS TRANSFER FROM THE DATA OF MEASUREMENTS IN NONSTATIONARY REGIMES

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For a number of laws governing the variation of the characteristics of internal heat and mass transfer with respect to a spatial variable, we derive explicit functions relating them to the results of measurements of nonstationary temperatures or other potentials.

Many physical processes can be described by partial differential equations of the type represented by the nonstationary heat-conduction equation with coefficients which depend on a spatial variable. It is therefore of great practical interest to construct effective calculation algorithms whereby the data of measurements of some transfer potentials (for example, temperatures) can be used for estimating the parameters determining the character of the spatial variation of the coefficients involved. In some cases, exact explicit functions sufficiently suitable for practical realization can be obtained by using the method employed in [1, 2], namely, an analysis of the properties of the analytic solutions of the problem in the space of Laplace mappings.

In the case when it is permissible to describe a real process by a one-dimensional parabolic operator with coefficients dependent on a spatial variable, of the form

$$r^{-k}\frac{\partial}{\partial r}\lambda(r)\,r^k\frac{\partial T(r,\ \tau)}{\partial r}=c(r)\frac{\partial T(r,\ \tau)}{\partial \tau}, \tag{1}$$

where k = 0, 1, 2 for plane, cylindrical, and spherical fields, respectively, it is possible for a number of specific laws of variation of λ and c to obtain exact analytic solutions [3]. In particular, in the space of Laplace mappings a solution of the form

$$T(r,\ s)=\left(\frac{\lambda r^k c r^k}{E^2}\right)^{\frac{1}{2(m-2)}}\left\{AI_{-\frac{1}{m}}\left[\frac{2E\sqrt{s}}{m}\left(\frac{\lambda r^k c r^k}{E^2}\right)^{\frac{m}{2(m-2)}}\right]+BK_{-\frac{1}{m}}\left[\frac{2E\sqrt{s}}{m}\left(\frac{\lambda r^k c r^k}{E^2}\right)^{\frac{m}{2(m-2)}}\right]\right\} \tag{2}$$

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